Formalizing mathematics with Lean and Mathlib

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Lean Seminar at Rutgers

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Why Formalize Mathematics?

- In principle, mathematics is formal: all definition, theorems, proofs can be written out explicitly in a way that can be read by anyone.
- In practice, mathematicians write articles that are a tool of communication. They require pre-existing knowledge, and contain logical leaps.
- Communication is complex and is not, in practice, completely formal.
- Risks of error: through miscommunications, misunderstandings, errors in logic.
- Solution: computer-based proof assistants.

Proof assistants

- The dream: software that interprets human-readable papers, formalizes them, and verifies them.
- First step: software that verifies already formalized proofs.
- Not a new idea: Isabelle/HOL (1986), Rocq/Coq (1989)
- Successes:
  - Four color theorem: [Gonthier, 2005] (Coq)
  - Kepler conjecture: [Hales, 2014] (HOL)
  - ...
- Here: Lean (2013).

Lean

- Developed by Leonardo de Moura, starting in 2013.
- Stable versions: Lean 3 (2017), Lean 4 (2021).
- Ideas: small kernel, readable proofs.
- Mathlib: Repository of formalized and verified theorems.
- Has been used for research-level mathematics: Liquid tensor experiment (Scholze et al).

How to use Lean?

- Local install: install `elan`, `lake`.
- Online: https://live.lean-lang.org/