A Renormalization Group based framework to study twisted bilayer graphene

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joint with **Vieri Mastropietro**

Dedicated to the memory of Giosi Benfatto,
whose work is the foundation on which these results are built.

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Kondo Effect in a Fermionic Hierarchical Model

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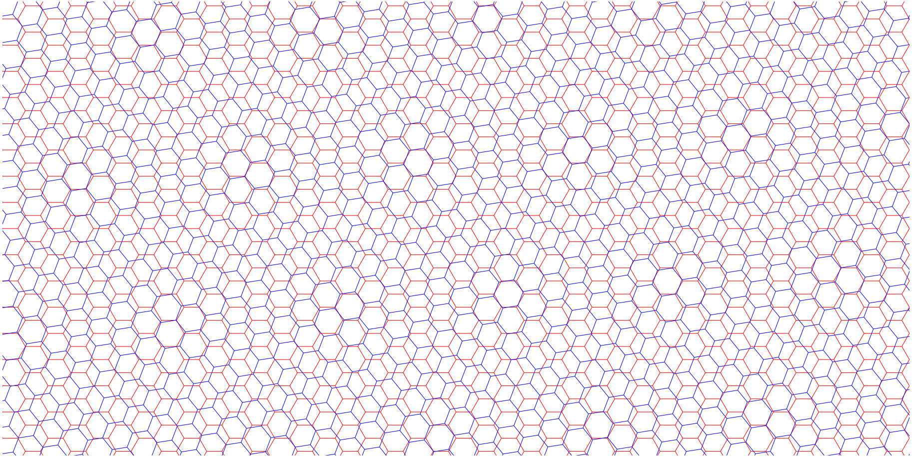
Twisted Bilayer Graphene



Twisted Bilayer Graphene

- Two graphene sheets at an angle θ.
- Theoretically studied: [Bistritzer, MacDonald, 2011]
- Experimental realization: [Cao, Fatemi, Fang, Watanabe, Taniguchi, Kaxiras, Jarillo-Herrero, 2018]
- At certain specific angles ("magic angles"): flat bands, leading to "unconventional" superconductivity.
- First (largest) Magic Angle: $\approx 1.05°$

Model

- Graphene lattices: $\mathcal{L}_1$ and $\mathcal{L}_2$.
- Intra-layer: for $i = 1, 2$,

$$H_i = \sum_{x \sim y \in \mathcal{L}_i} c_{i,x}^\dagger c_{i,y}$$

- Inter-layer:

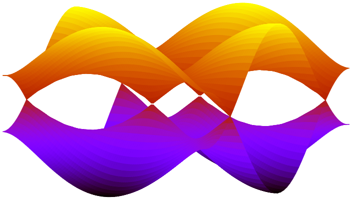
$$V = \lambda \sum_{x \in \mathcal{L}_1} \sum_{y \in \mathcal{L}_2} \phi(x - y)(c_{1,x}^\dagger c_{2,y} + c_{2,y}^\dagger c_{1,x})$$

- Total Hamiltonian

$$H = H_1 + H_2 + V.$$

Intra-layer model

- If $\lambda = 0$, $H = H_1 + H_2$: two independent graphene layers.
- Hamiltonian is diagonalizable in Fourier space.



- Fermi points: singularities of two-point correlation.

Main result

- **Theorem**: If ϕ is short-ranged, and $\hat{\phi}(q) \leqslant ce^{-\kappa|q|}$ (plus a technical assumption), then, for any $[\theta_0, \theta_1] \subset [0, 2\pi)$ and any $C_0 > 0$, there exists a set of θ's that has large measure (its complement has measure at most $(C_0/(\theta_0 - \theta_1)^2)$) for which (upon adding an appropriate counter-term to the Hamiltonian) the Schwinger function is close to the intra-layer one near the Fermi points:

$$S_i(\mathbf{k} + \mathbf{p}_{F,i}) = \begin{pmatrix} -iZ_{j,\omega}k_0 & (iv_{i,\omega}k_1 - w_{i,\omega}\omega k_2) \\ (-iv^*_{i,\omega}k_1 - w^*_{i,\omega}\omega k_2) & -iZ_{j,\omega}k_0 \end{pmatrix}^{-1} (1 + O(\mathbf{k}^2))$$

$$Z_{j,\omega} = 1 + O(\lambda) \in \mathbb{R}, \quad v_{i,\omega}, w_{i,\omega} = \frac{3}{2} + O(\lambda) \in \mathbb{C}$$

(Counter term)

- The counter-term fixes the Fermi points. Without it, they would be shifted in momentum space.
- Form

$$\begin{pmatrix} 0 & \nu_{i,\omega} \\ \nu_{i,\omega}^* & 0 \end{pmatrix}$$

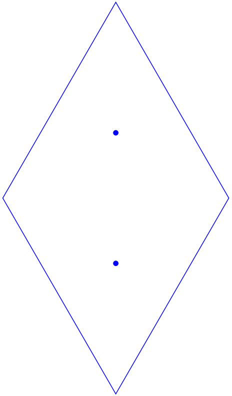
with

$$\nu = O(\lambda) \in \mathbb{C}.$$

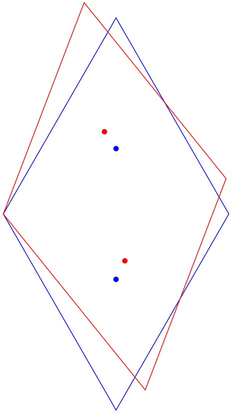
Renormalization group

- Renormalization group analysis: multiscale perturbation theory in λ.
- Main obstacle: the Fermi points are effectively dense (small divisor problem).

Small divisor problem



Small divisor problem



Small divisor problem

- The dual lattice $\hat{\mathcal{L}}_1$ is periodic modulo $\mathbb{Z}b_1 + \mathbb{Z}b_2$. On the other hand $\hat{\mathcal{L}}_2$ is periodic modulo $\mathbb{Z}b'_1 + \mathbb{Z}b'_2$ with $b'_i = R_\theta b_i$ (b'_i is rotated by an angle θ).
- Small divisor: for $l, m \in \mathbb{Z}^2$

$$|q_{i,j,\omega,\omega',l,m}| := |p^{\omega}_{F,i} - p^{\omega'}_{F,j} + l_1 b_1 + l_2 b_2 + m_1 b'_1 + m_2 b'_2| \ll 1$$

  leads to a divergence.
- However, such terms come with $\hat{\phi}(|q_{i,j,\omega,\omega',l,m}|)$, which decays exponentially with l, m.
- So we must ensure that for $|q_{i,j,\omega,\omega',l,m}|$ to be small, l, m have to be large enough to be dampened by the exponential decay of $\hat{\phi}$.

Diophantine condition

- For $i = j$, $\omega = \omega'$: choose θ such that

$$|l_1 b_1 + l_2 b_2 + m_1 R_\theta b_1 + m_2 R_\theta b_2| \geqslant \frac{C_0}{|l|^\tau}, \frac{C_0}{|m|^\tau}$$

- Complication: this is a two-dimensional condition, but there is only one parameter θ.
- The set of such θ's has arbitrarily high measure (at the price of decreasing C_0).

Renormalization group analysis

- Remainder of the analysis: similar to RG for quasi-periodic potentials:
  - [Benfatto, Gentile, Mastropietro, 1997]
  - [Mastropietro, 2017]
  - [Gallone, Mastropietro, 2024].

Outlook

- Bistritzer-MacDonald model: only keeps 3 values of l, m.
- Our construction yields a framework to study twisted bilayers using absolutely convergent series.
- Next step: compute the first few orders and get a handle on the Magic Angles.