

Liquid crystals and the Heilmann-Lieb model

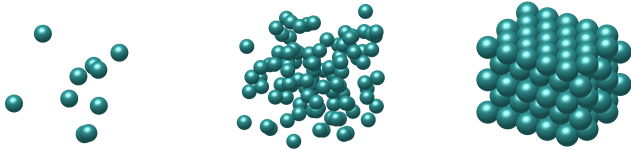
Ian Jauslin

joint with **Elliott H. Lieb**

arXiv: 1709.05297

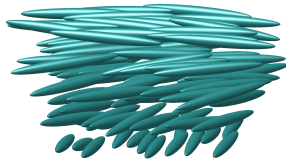
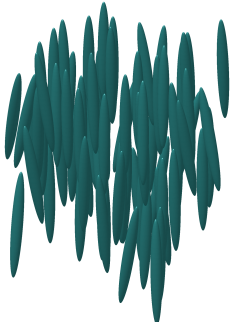
<http://ian.jauslin.org>

Gas-liquid-crystal



Liquid crystals

- Orientational order and positional disorder.

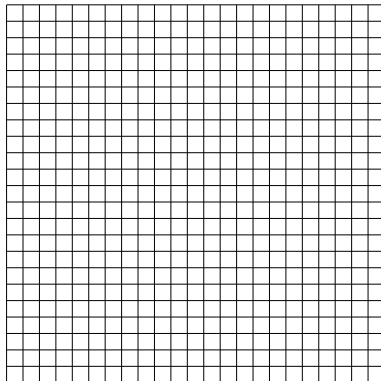


History

- [Onsager, 1949]: mean field model for hard needles in $\mathbb{R}^3$.
- [Heilmann, Lieb, 1979]: interacting dimers.

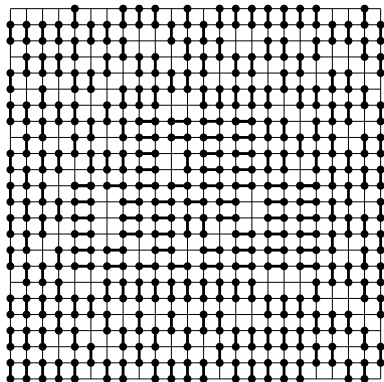
Heilmann-Lieb model

[Heilmann, Lieb, 1979]



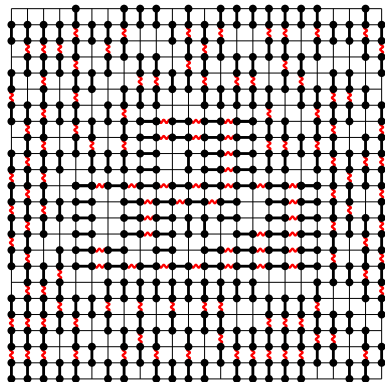
Heilmann-Lieb model

[Heilmann, Lieb, 1979]



Heilmann-Lieb model

[Heilmann, Lieb, 1979]



Heilmann-Lieb conjecture

- [Heilmann, Lieb, 1979]: proved orientational order using reflection positivity.
- HL Conjecture: absence of positional order.

History

- [Onsager, 1949]: mean field model for hard needles in $\mathbb{R}^3$.
- [Heilmann, Lieb, 1979]: interacting dimers.
- [Bricmont, Kuroda, Lebowitz, 1984]: hard needles in $\mathbb{R}^2$ with a *finite* number of orientations.
- [Ioffe, Velenik, Zahradník, 2006]: hard rods in $\mathbb{Z}^2$ (variable length).
- [Disertori, Giuliani, 2013]: hard rods in $\mathbb{Z}^2$.
- [Disertori, Giuliani, Jauslin, 2019]: hard plates in $\mathbb{Z}^3$.

Heilmann-Lieb conjecture

- [Heilmann, Lieb, 1979]: proved orientational order using reflection positivity.
- HL Conjecture: absence of positional order.
- [Alberici, 2016]: different fugacities for horizontal and vertical dimers.
- [Papanikolaou, Charrier, Fradkin, 2014]: numerics.

Heilmann-Lieb model

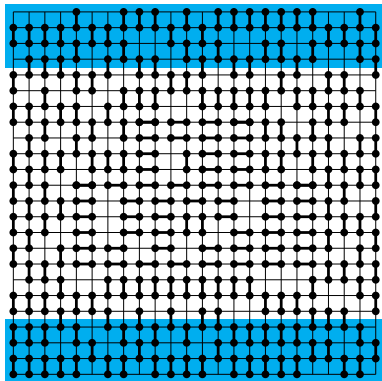
- Grand-canonical Gibbs measure:

$$\langle A \rangle_v := \lim_{\Lambda \rightarrow \mathbb{Z}^2} \frac{1}{\Xi_v(\Lambda)} \sum_{\underline{\delta} \in \Omega_v(\Lambda)} A(\underline{\delta}) z^{|\underline{\delta}|} \prod_{\delta \neq \delta' \in \underline{\delta}} e^{\frac{1}{2} J \mathbb{1}_{\delta \sim \delta'}}$$

- ▶ Λ : finite box.
- ▶ $\Omega_v(\Lambda)$: non-overlapping dimer configurations satisfying the boundary condition.
- ▶ $z \geq 0$: fugacity.
- ▶ $J \geq 0$: interaction strength.
- ▶ $\mathbb{1}_{\delta \sim \delta'}$ indicator that dimers are adjacent and aligned.

Boundary condition

- Fix length $\ell_0 := e^{\frac{3}{2}J} \sqrt{z}$,



Theorem

For $1 \ll z \ll J$, $\|(x, y)\|_{\text{HL}} := J|x| + e^{-\frac{3}{2}J} z^{-\frac{1}{2}}|y|$,

- Given two vertical edges e_v, f_v , $\langle \mathbb{1}_{e_v} \rangle_v$ is *independent* of e_v and

$$\langle \mathbb{1}_{e_v} \rangle_v = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J} z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_v} \mathbb{1}_{f_v} \rangle_v - \langle \mathbb{1}_{e_v} \rangle_v \langle \mathbb{1}_{f_v} \rangle_v = O(e^{-c \text{ dist}_{\text{HL}}(e_v, f_v)})$$

- Given two horizontal edges e_h, f_h , $\langle \mathbb{1}_{e_h} \rangle_v$ is *independent* of e_h and

$$\langle \mathbb{1}_{e_h} \rangle_v = O(e^{-3J})$$

$$\langle \mathbb{1}_{e_h} \mathbb{1}_{f_h} \rangle_v - \langle \mathbb{1}_{e_h} \rangle_v \langle \mathbb{1}_{f_h} \rangle_v = O(e^{-6J - c \text{ dist}_{\text{HL}}(e_h, f_h)})$$

1D system

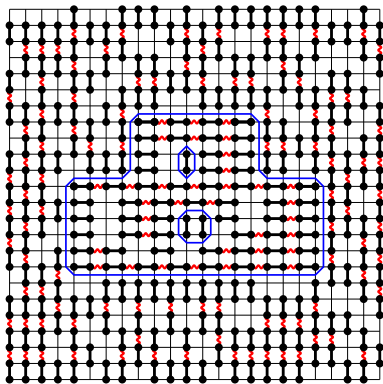
- *Only* vertical dimers: integrable.
- Given two vertical edges e_v, f_v , $\langle \mathbb{1}_{e_v} \rangle_v$ is *independent* of e_v and

$$\langle \mathbb{1}_{e_v} \rangle_v = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J} z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_v} \mathbb{1}_{f_v} \rangle_v - \langle \mathbb{1}_{e_v} \rangle_v \langle \mathbb{1}_{f_v} \rangle_v = O(e^{-c \operatorname{dist}_{1D}(e_v, f_v)})$$

with $\|(x, y)\|_{1D} := e^{-\frac{3}{2}J} z^{-\frac{1}{2}} |y|$.

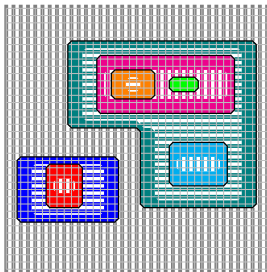
Loop model



- Weight of a loop of length $|l|$: $e^{-\frac{1}{2}J|l|}$.

Difficulty: loops interact

- Correlated dimers induce an interaction between loops, which decays exponentially with a rate $e^{-\frac{3}{2}J}z^{-\frac{1}{2}}$.



- Vertical-to-horizontal boundaries and horizontal-to-vertical ones have different geometries.

Parameter regime

