Exact solution of the time dependent Schrödinger equation for photoemission from a metal surface

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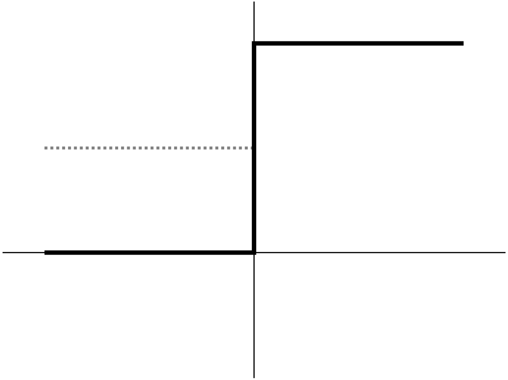
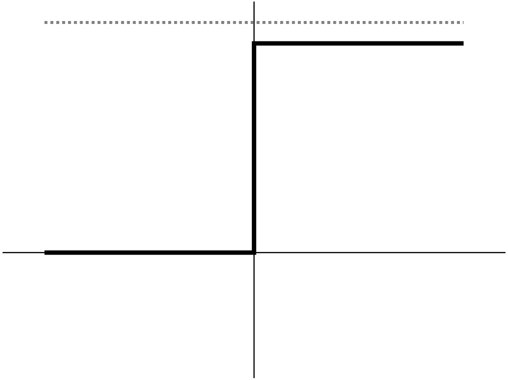
Electron emission

$$V(x) = U\Theta(x), \quad E_{\mathrm{F}} = k_{\mathrm{F}}^2 < U$$



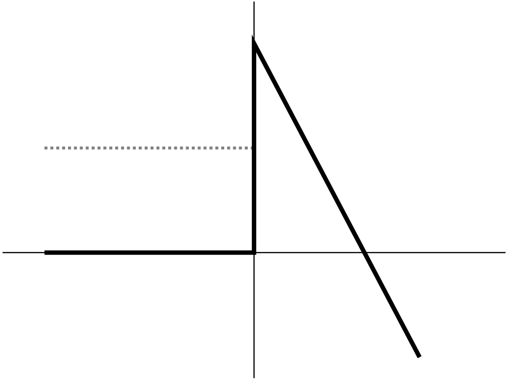
Thermal emission

$$V(x) = U\Theta(x), \quad k^2 > U$$



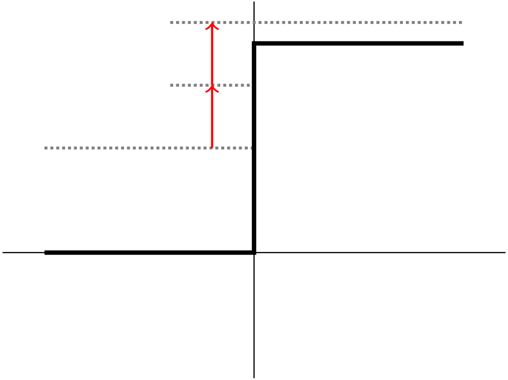
Field emission

$$V(x) = \Theta(x)(U - Ex)$$



Photoemission

$$V_t(x) = \Theta(x)(U - E_t x), \quad E_t = 2\epsilon\omega\cos(\omega t)$$



Photoemission

- Photoelectric effect: early observations: [Hertz, 1887], [Hallwachs, 1888], [Lenard, 1900].
- When a metal is irradiated with ultra-violet light, electrons are ionized, with kinetic energies at integer multiples of $\hbar\omega$.
- [Einstein, 1905]: interpretation: quanta of light (*photons*) of energy $\hbar\omega$ are absorbed by the electrons, whose kinetic energy is raised by $n\hbar\omega$, and can escape the metal.

Photoemission

- Time dependent potential:

$$V_t(x) = \Theta(x)(U - 2\epsilon\omega\cos(\omega t)x)$$

- Schrödinger equation

$$i\partial_t\psi(x,t) = -\Delta\psi(x,t) + V_t(x)\psi(x,t)$$

- Magnetic gauge:

$$\Psi(x,t) := \psi(x,t)e^{-ix\Theta(x)A(t)}, \quad A(t) := \int_0^t ds\ 2\epsilon\omega\cos(\omega s) = 2\epsilon\sin(\omega t)$$

satisfies

$$i\partial_t\Psi(x,t) = \big((-i\nabla + \Theta(x)A(t))^2 + \Theta(x)U\big)\,\Psi(x,t)$$

Periodic solution

- [Faisal, Kamiński, Saczuk, 2005]

$$\Psi_{\mathrm{FKS}}(x,t) = \begin{cases} e^{ikx}\exp\left(-ik^2 t\right) + \Psi_R(x,t) & \text{if } x < 0 \\ \Psi_T(x,t) & \text{if } x > 0 \end{cases}$$

$$\Psi_R(x,t) = \sum_{M\in\mathbb{Z}} R_M e^{-iq_M x}\exp\left(-iq_M^2 t\right), \quad q_M = \sqrt{k^2 + M\omega}$$

$$\Psi_T(x,t) = \sum_{M\in\mathbb{Z}} T_M e^{ip_M x}\exp\left(-iUt - i\int_0^t d\tau\ (p_M - A(\tau))^2\right)$$

$$p_M = \sqrt{k^2 - U + M\omega - 2\epsilon^2}$$

- $\Psi(x,t)$, $(-i\nabla + \Theta(x)A(t))\Psi(x,t)$ are continuous.

Initial value problem

- Initial condition:

$$\Psi(x,0) = \begin{cases} e^{ikx} + R_0 e^{-ikx} & x < 0 \\ T_0 e^{-\sqrt{U-k^2}x} & x > 0 \end{cases}$$

  R_0 and T_0 ensure that Ψ and $\partial\Psi$ are continuous.

- In progress: $\Psi(x,t)$ behaves asymptotically like Ψ_{FKS}:

$$\psi(x,t) = \psi_{\mathrm{FKS}}(x,t) + \left(\frac{t}{\tau_{\mathrm{FKS}}(x)}\right)^{-\frac{3}{2}} + O(t^{-\frac{5}{2}}).$$