

# Liquid crystals and the Heilmann-Lieb model

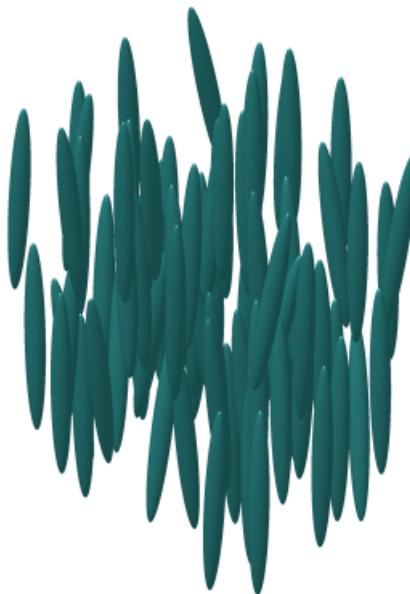
Ian Jauslin

joint with **Elliott H. Lieb**

# Liquid crystals

---

- Orientational order and positional disorder.



## Models

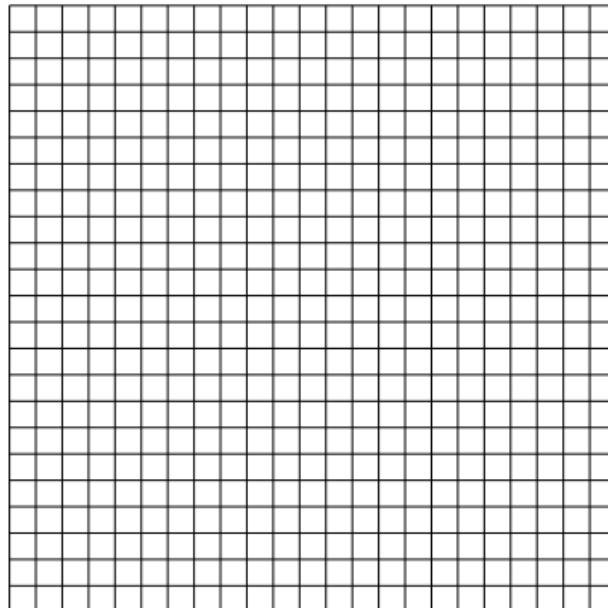
---

- [Heilmann, Lieb, 1979]: interacting dimers.
- [Bricmont, Kuroda, Lebowitz, 1984]: hard needles in  $\mathbb{R}^2$ .
- [Ioffe, Velenik, Zahradník, 2006]: hard rods in  $\mathbb{Z}^2$  (variable length).
- [Disertori, Giuliani, 2013]: hard rods in  $\mathbb{Z}^2$ .
- [Disertori, Giuliani, Jauslin, 2018]: hard plates in  $\mathbb{R}^3$ .

# Heilmann-Lieb model

---

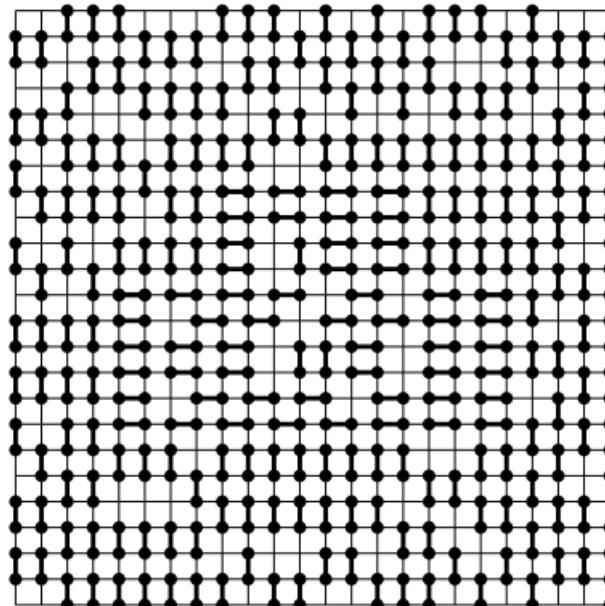
[Heilmann, Lieb, 1979]



# Heilmann-Lieb model

---

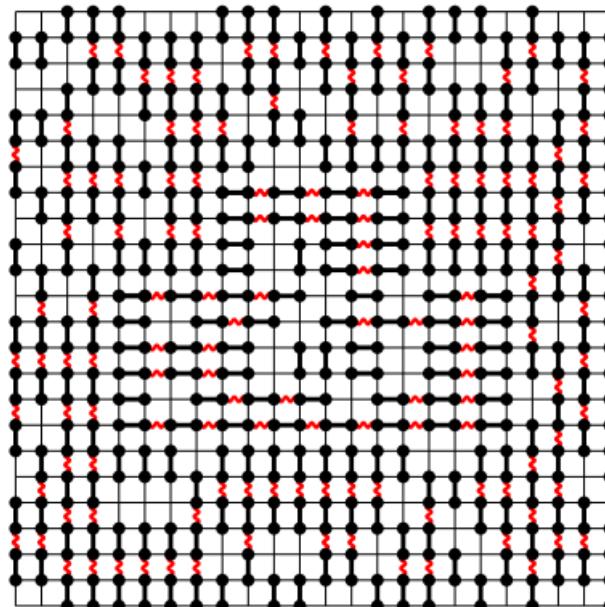
[Heilmann, Lieb, 1979]



# Heilmann-Lieb model

---

[Heilmann, Lieb, 1979]



# Heilmann-Lieb model

---

- Grand-canonical Gibbs measure:

$$\langle A \rangle_v := \lim_{\Lambda \rightarrow \mathbb{Z}^2} \frac{1}{\Xi_{\Lambda, v}(z)} \sum_{\underline{\delta} \in \Omega_v(\Lambda)} A(\underline{\delta}) z^{|\underline{\delta}|} \prod_{\delta \neq \delta' \in \underline{\delta}} e^{\frac{1}{2} J \mathbb{1}_{\delta \sim \delta'}}$$

- ▶  $\Lambda$ : finite box.
- ▶  $\Omega_v(\Lambda)$ : non-overlapping dimer configurations satisfying the boundary condition.
- ▶  $z \geq 0$ : fugacity.
- ▶  $J \geq 0$ : interaction strength.
- ▶  $\mathbb{1}_{\delta \sim \delta'}$  indicator that dimers are adjacent and aligned.

## Theorem

---

For  $1 \ll z \ll J$ ,  $\|(x, y)\|_{\text{HL}} := J|x| + e^{-\frac{3}{2}J}z^{-\frac{1}{2}}|y|$ ,

- Given two vertical edges  $e_v, f_v$ ,  $\langle \mathbb{1}_{e_v} \rangle_v$  is *independent* of  $e_v$  and

$$\langle \mathbb{1}_{e_v} \rangle_v = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J}z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_v} \mathbb{1}_{f_v} \rangle_v - \langle \mathbb{1}_{e_v} \rangle_v \langle \mathbb{1}_{f_v} \rangle_v = O(e^{-c \text{ dist}_{\text{HL}}(e_v, f_v)})$$

- Given two horizontal edges  $e_h, f_h$ ,  $\langle \mathbb{1}_{e_h} \rangle_v$  is *independent* of  $e_h$  and

$$\langle \mathbb{1}_{e_h} \rangle_v = O(e^{-3J})$$

$$\langle \mathbb{1}_{e_h} \mathbb{1}_{f_h} \rangle_v - \langle \mathbb{1}_{e_h} \rangle_v \langle \mathbb{1}_{f_h} \rangle_v = O(e^{-3J - c \text{ dist}_{\text{HL}}(e_v, f_v)})$$

# 1D system

---

- [Heilmann, Lieb, 1979]: mostly vertical dimers.
- *Only* vertical dimers: integrable.
- Given two vertical edges  $e_v, f_v$ ,  $\langle \mathbb{1}_{e_v} \rangle_v$  is *independent* of  $e_v$  and

$$\langle \mathbb{1}_{e_v} \rangle_v = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J} z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_v} \mathbb{1}_{f_v} \rangle_v - \langle \mathbb{1}_{e_v} \rangle_v \langle \mathbb{1}_{f_v} \rangle_v = O(e^{-c \text{ dist}_{1D}(e_v, f_v)})$$

with  $\|(x, y)\|_{1D} := e^{-\frac{3}{2}J} z^{-\frac{1}{2}} |y|$ .