

High density phases of hard-core lattice particle systems

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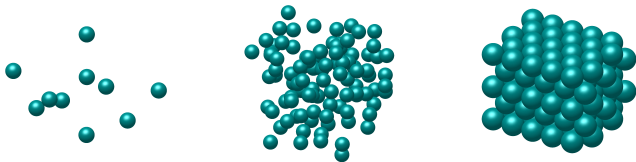
joint with **Joel L. Lebowitz** and **Elliott H. Lieb**

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Gas-liquid-crystal

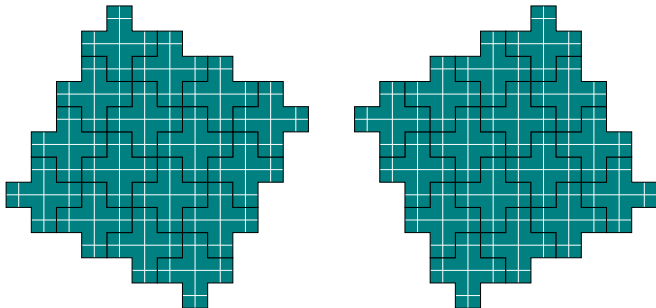


Hard-core lattice particle (HCLP) systems



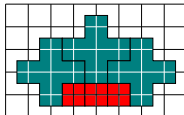
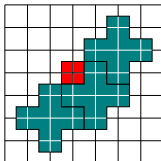
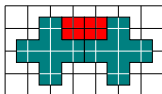
Non-sliding HCLPs

- There exist a **finite** number τ of tilings which are **periodic** and **isometric** to each other.



Non-sliding HCLPs

- Defects are **localized**: for every connected particle configuration X that is *not* the subset of a close packing and every $Y \supset X$, there is empty space in Y neighboring X .



Observables

- Gibbs measure:

$$\langle A \rangle_\nu := \lim_{\Lambda \rightarrow \Lambda_\infty} \frac{1}{\Xi_{\Lambda, \nu}(z)} \sum_{X \subset \Lambda} A(X) z^{|X|} \mathfrak{B}_\nu(X) \prod_{x \neq x' \in X} \varphi(x, x')$$

- ▶ Λ : finite subset of lattice Λ_∞ .
 - ▶ $z \geq 0$: fugacity.
 - ▶ $\varphi(x, x')$: hard-core interaction.
 - ▶ $\mathfrak{B}_\nu$: boundary condition: favors the ν -th tiling.
- Pressure:

$$p(z) := \lim_{\Lambda \rightarrow \Lambda_\infty} \frac{1}{|\Lambda|} \log \Xi_{\Lambda, \nu}(z).$$

Theorem

- $p(z) - \rho_m \log z$ and $\langle \mathbb{1}_{x_1} \cdots \mathbb{1}_{x_n} \rangle_\nu$ are **analytic** functions of $1/z$ for large values of z .
- There are τ distinct Gibbs states:

$$\langle \mathbb{1}_x \rangle_\nu = \begin{cases} 1 + O(y) & \text{if } x \in \mathcal{L}_\nu \\ O(y) & \text{if not.} \end{cases}$$

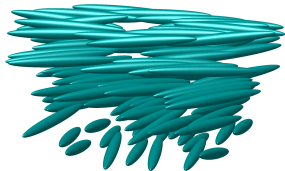
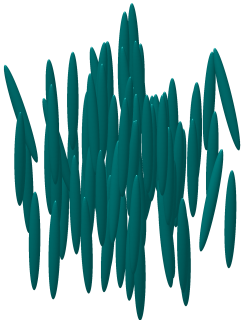
High-fugacity expansion

$$p(y) = -\rho_m \log y + \sum_{k=1}^{\infty} c_k y^k$$

- [Gaunt, Fisher, 1965]: diamonds: for $k \leq 9$.
- [Joyce, 1988]: hexagons (integrable, [Baxter, 1980]).
- [Eisenberg, Baram, 2005]: crosses: for $k \leq 6$.
- For sliding models, the high-fugacity expansion is ill-defined.

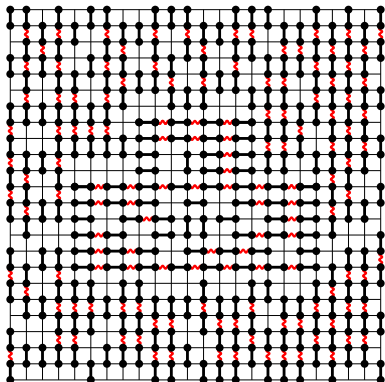
Liquid crystals

- Orientational order and positional disorder.



Heilmann-Lieb model

[Heilmann, Lieb, 1979]



Heilmann-Lieb model

- Gibbs measure:

$$\langle A \rangle_v := \lim_{\Lambda \rightarrow \mathbb{Z}^2} \frac{1}{\Xi_{\Lambda, v}(z)} \sum_{\underline{\delta} \in \Omega_v(\Lambda)} A(\underline{\delta}) z^{|\underline{\delta}|} \prod_{\delta \neq \delta' \in \underline{\delta}} e^{\frac{1}{2} J \mathbb{1}_{\delta \sim \delta'}}$$

- ▶ Λ : finite box.
- ▶ $\Omega_v(\Lambda)$: non-overlapping dimer configurations satisfying the boundary condition.
- ▶ $z \geq 0$: fugacity.
- ▶ $J \geq 0$: interaction strength.
- ▶ $\mathbb{1}_{\delta \sim \delta'}$ indicator that dimers are adjacent and aligned.

Heilmann-Lieb conjecture

- [Heilmann, Lieb, 1979]: proved orientational order using reflection positivity.
- HL Conjecture: absence of positional order.
- [Ioffe, Velenik, Zahradník, 2006], [Disertori, Giuliani, 2013]: nematic liquid crystal phase in systems of hard rods on $\mathbb{Z}^2$.
- [Alberici, 2016]: different fugacities for horizontal and vertical dimers.
- [Papanikolaou, Charrier, Fradkin, 2014]: numerics.

Theorem

For $1 \ll z \ll J$, $\|(x, y)\|_{\text{HL}} := J|x| + e^{-\frac{3}{2}J} z^{-\frac{1}{2}}|y|$,

- Given two vertical edges e_v, f_v , $\langle \mathbb{1}_{e_v} \rangle_v$ is *independent* of e_v and

$$\langle \mathbb{1}_{e_v} \rangle_v = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J} z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_v} \mathbb{1}_{f_v} \rangle_v - \langle \mathbb{1}_{e_v} \rangle_v \langle \mathbb{1}_{f_v} \rangle_v = O(e^{-c \text{ dist}_{\text{HL}}(e_v, f_v)})$$

- Given two horizontal edges e_h, f_h , $\langle \mathbb{1}_{e_h} \rangle_v$ is *independent* of e_h and

$$\langle \mathbb{1}_{e_h} \rangle_v = O(e^{-3J})$$

$$\langle \mathbb{1}_{e_h} \mathbb{1}_{f_h} \rangle_v - \langle \mathbb{1}_{e_h} \rangle_v \langle \mathbb{1}_{f_h} \rangle_v = O(e^{-3J - c \text{ dist}_{\text{HL}}(e_h, f_h)})$$