High density phases of hard-core lattice particle systems

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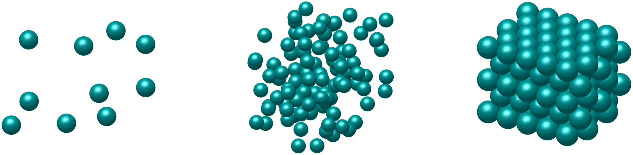
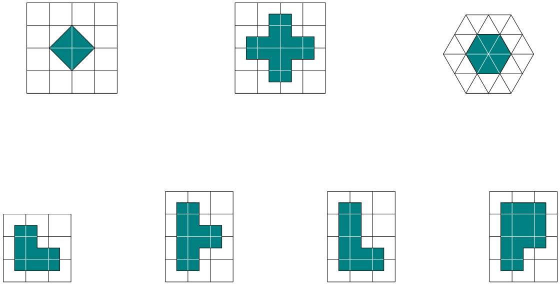
Gas-liquid-crystal

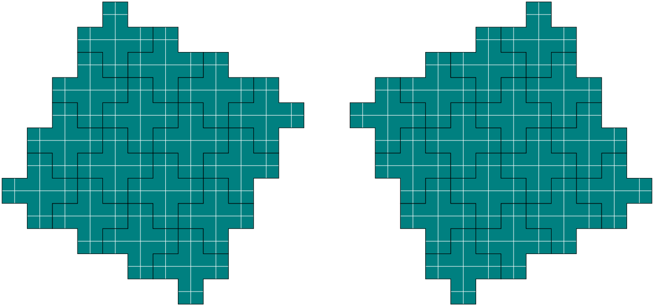


Hard-core lattice particle (HCLP) systems



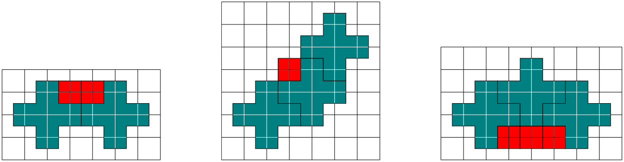
Non-sliding HCLPs

- There exist a **finite** number τ of tilings which are **periodic** and **isometric** to each other.



Non-sliding HCLPs

- Defects are **localized**: for every connected particle configuration X that is *not* the subset of a close packing and every $Y \supset X$, there is empty space in Y neighboring X.



Observables

- Gibbs measure:

$$\langle A\rangle_\nu := \lim_{\Lambda\to\Lambda_\infty} \frac{1}{\Xi_{\Lambda,\nu}(z)} \sum_{X\subset\Lambda} A(X) z^{|X|}\mathfrak{B}_\nu(X) \prod_{x\neq x'\in X} \varphi(x,x')$$

  - Λ: finite subset of lattice Λ_∞.
  - $z \geqslant 0$: fugacity.
  - $\varphi(x,x')$: hard-core interaction.
  - $\mathfrak{B}_\nu$: boundary condition: favors the ν-th tiling.
- Pressure:

$$p(z) := \lim_{\Lambda\to\Lambda_\infty} \frac{1}{|\Lambda|} \log \Xi_{\Lambda,\nu}(z).$$

Theorem

- $p(z) - \rho_m \log z$ and $\langle \mathbb{1}_{x_1} \cdots \mathbb{1}_{x_n} \rangle_\nu$ are **analytic** functions of $1/z$ for large values of z.

- There are τ distinct Gibbs states:

$$\langle \mathbb{1}_x \rangle_\nu = \begin{cases} 1 + O(y) & \text{if } x \in \mathcal{L}_\nu \\ O(y) & \text{if not.} \end{cases}$$

High-fugacity expansion

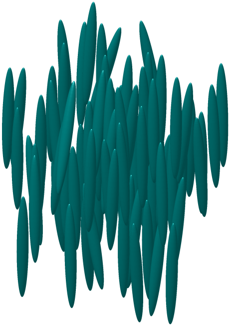
$$p(y) = -\rho_m \log y + \sum_{k=1}^{\infty} c_k y^k$$

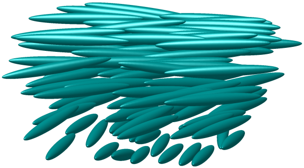
- [Gaunt, Fisher, 1965]: diamonds: for $k \leqslant 9$.
- [Joyce, 1988]: hexagons (integrable, [Baxter, 1980]).

- [Eisenberg, Baram, 2005]: crosses: for $k \leqslant 6$.
- For sliding models, the high-fugacity expansion is ill-defined.

Liquid crystals

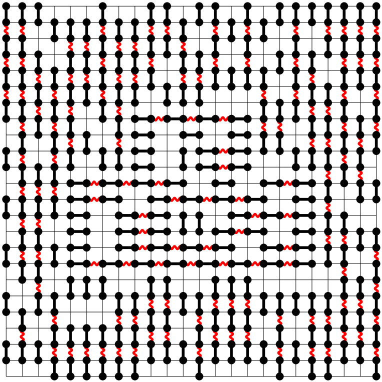
- Orientational order and positional disorder.





Heilmann-Lieb model

[Heilmann, Lieb, 1979]



Heilmann-Lieb model

- Gibbs measure:

$$\langle A\rangle_{\mathrm{v}} := \lim_{\Lambda\to\mathbb{Z}^2}\frac{1}{\Xi_{\Lambda,\mathrm{v}}(z)}\sum_{\underline{\delta}\in\Omega_{\mathrm{v}}(\Lambda)} A(\underline{\delta})z^{|\underline{\delta}|}\prod_{\delta\neq\delta'\in\underline{\delta}} e^{\frac{1}{2}J\mathbb{1}_{\delta\sim\delta'}}$$

  - Λ: finite box.
  - $\Omega_{\mathrm{v}}(\Lambda)$: non-overlapping dimer configurations satisfying the boundary condition.
  - $z \geqslant 0$: fugacity.
  - $J \geqslant 0$: interaction strength.
  - $\mathbb{1}_{\delta\sim\delta'}$ indicator that dimers are adjacent and aligned.

Heilmann-Lieb conjecture

- [Heilmann, Lieb, 1979]: proved orientational order using reflection positivity.
- HL Conjecture: absence of positional order.
- [Ioffe, Velenik, Zahradník, 2006], [Disertori, Giuliani, 2013]: nematic liquid crystal phase in systems of hard rods on $\mathbb{Z}^2$.
- [Alberici, 2016]: different fugacities for horizontal and vertical dimers.
- [Papanikolaou, Charrier, Fradkin, 2014]: numerics.

Theorem

For $1 \ll z \ll J$, $\|(x,y)\|_{\mathrm{HL}} := J|x| + e^{-\frac{3}{2}J} z^{-\frac{1}{2}} |y|$,

- Given two vertical edges $e_{\mathrm{v}}, f_{\mathrm{v}}$, $\langle \mathbb{1}_{e_{\mathrm{v}}} \rangle_{\mathrm{v}}$ is *independent* of e_{v} and

$$\langle \mathbb{1}_{e_{\mathrm{v}}} \rangle_{\mathrm{v}} = \frac{1}{2}(1 + O(e^{-\frac{1}{2}J} z^{-\frac{1}{2}}))$$

$$\langle \mathbb{1}_{e_{\mathrm{v}}} \mathbb{1}_{f_{\mathrm{v}}} \rangle_{\mathrm{v}} - \langle \mathbb{1}_{e_{\mathrm{v}}} \rangle_{\mathrm{v}} \langle \mathbb{1}_{f_{\mathrm{v}}} \rangle_{\mathrm{v}} = O(e^{-c \ \mathrm{dist}_{\mathrm{HL}}(e_{\mathrm{v}}, f_{\mathrm{v}})})$$

- Given two horizontal edges $e_{\mathrm{h}}, f_{\mathrm{h}}$, $\langle \mathbb{1}_{e_{\mathrm{h}}} \rangle_{\mathrm{v}}$ is *independent* of e_{h} and

$$\langle \mathbb{1}_{e_{\mathrm{h}}} \rangle_{\mathrm{v}} = O(e^{-3J})$$

$$\langle \mathbb{1}_{e_{\mathrm{h}}} \mathbb{1}_{f_{\mathrm{v}}} \rangle_{\mathrm{v}} - \langle \mathbb{1}_{e_{\mathrm{h}}} \rangle_{\mathrm{v}} \langle \mathbb{1}_{f_{\mathrm{h}}} \rangle_{\mathrm{v}} = O(e^{-3J - c \ \mathrm{dist}_{\mathrm{HL}}(e_{\mathrm{v}}, f_{\mathrm{v}})})$$