# Emergence of a nematic phase in a system of hard plates

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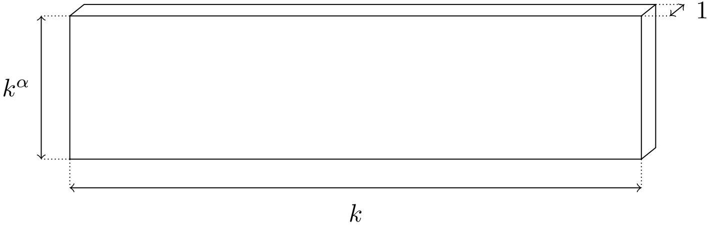
# Hard plates

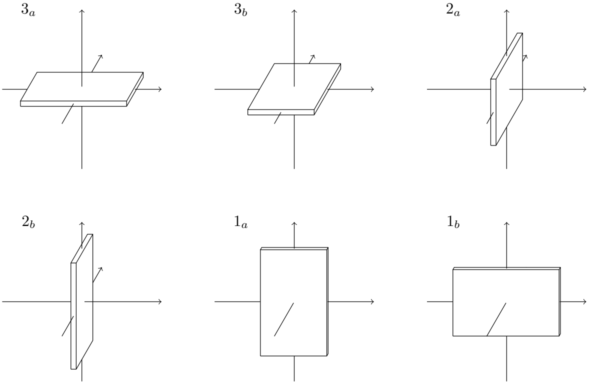
- Parallelepiped $k \times k^\alpha \times 1$, $\alpha \in [0,1]$, $k \gg 1$




- Center in $\mathbb{R}^3$

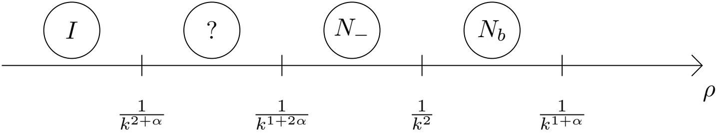
# Hard plates

- 6 orientations



# Heuristics

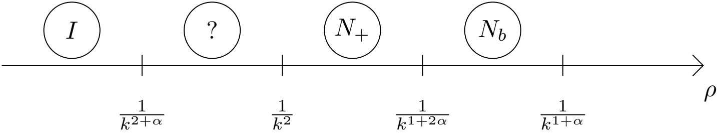
- For $\frac{1}{2} \leqslant \alpha \leqslant 1$




- $N_b$: biaxial nematic
- $N_-$: plate-like nematic
- $I$: isotropic
- ?: ?

# Heuristics

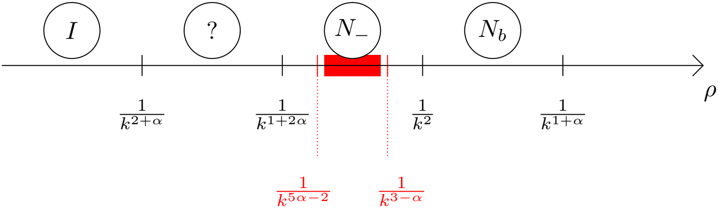
- For $0 \leqslant \alpha \leqslant \frac{1}{2}$




- $N_b$: biaxial nematic
- $N_+$: rod-like nematic
- $I$: isotropic
- ?: ?

# Result

- $\frac{5}{6} < \alpha < 1$ (we can also do $\alpha = 1$).



# Previous results

- [Disertori, Giuliani, 2013]: 2-dimensional hard rods (on a lattice).
- [Bricmont, Kuroda, Lebowitz, 1984]: 2-dimensional hard needles.
- [Onsager, 1949]: elongated molecules in 3 dimensions: 1st order phase transition! (non-rigorous)

# Setup

- *Type* of a plate: $q \in \{1, 2, 3\}$.
- (Grand-canonical) Gibbs average, in a box $\Lambda$, with $q$-boundary conditions:

$$\langle A \rangle_{\Lambda,q} := \frac{1}{Z(\Lambda|q)} \int_{\Omega_{\Lambda,q}} dP \; z^{|P|} \varphi(P) A(P)$$

  - $\Omega_{\Lambda,q}$: plate configurations with $q$-boundary conditions, $|P|$: number of plates in $P$,
  - $z$: *activity*: $z = e^{\beta\mu}$,
  - $\varphi(P)$: hard-core potential,
  - $Z(\Lambda|q)$: partition function (normalization),
  - $A$: *local* observable.

# Setup

- Boundary condition: any plate, centered at $x \in \Lambda$, that satisfies

$$d_\infty(x,\ \mathbb{R}^3 \setminus \Lambda) \leqslant \left(\frac{4}{1-\alpha} + 3\right)\frac{k}{2}$$

is of type $q$.

- Local observable:

$$A(P) = \sum_{p \in P} a(p)$$

and $a$ is *compactly supported*.

# Theorem

- If $zk^{3-\alpha} \ll 1 \ll zk^{5\alpha-2}$
  - $\mathbb{1}_x(P)$: indicator that $\exists p \in P$: $d_\infty(x,p) \leqslant \frac{1}{2}$:
    $$\langle \mathbb{1}_x(P) \rangle_{\Lambda,q} \equiv \rho = z(1+o(1))$$
  - $\mathcal{N}_{x,q}(P)$: number of plates $p \in P$ of type $q$ with $d_\infty(p,x) \leqslant \frac{k}{4}$: for $m \neq q$,
    $$\langle \mathcal{N}_{x,q}(P) \rangle_{\Lambda,q} \geqslant Czk^3 \gg 1, \quad \langle \mathcal{N}_{x,m}(P) \rangle_{\Lambda,q} = o(1)$$
  - There exists $\eta_k \to 0$ as $k \to \infty$ such that
    $$\langle \mathbb{1}_x(P); \mathbb{1}_y(P) \rangle^T_{\Lambda,q} \leqslant C\rho^2 \eta_k^{\frac{|x-y|}{k}}$$

# Cluster expansion

- For $\mathbf{A} \equiv (A_1, \cdots, A_n)$, $\mathbf{s} \equiv (s_1, \cdots, s_n) \in \mathbb{R}^n$

$$F_{\Lambda,q}(\mathbf{s} \cdot \mathbf{A}) := \log \int_{\Omega_{\Lambda,q}} dP \; z^{|P|} \varphi(P) e^{\sum_{i=1}^{n} s_i A_i(P)}$$

- Generating function:

$$\langle A_1, \cdots, A_n \rangle^T_{\Lambda,q} = \partial_{s_1} \cdots \partial_{s_n} F_{\Lambda,q}(\mathbf{s} \cdot \mathbf{A})\big|_{\mathbf{s}=0}$$

# Cluster expansion

$$F_{\Lambda,q}(\mathbf{s}\cdot\mathbf{A}) = F^{(0)}_{\Lambda,q}(\mathbf{s}\cdot\mathbf{A}) + \sum_{\mathcal{X}\in\Xi(\Lambda)} \phi^T(\mathcal{X}) \prod_{X\in\mathcal{X}} K^{(\mathbf{s}\cdot\mathbf{A})}_{\Lambda,q}(X)$$

- $F^{(0)}_{\Lambda,q}(\mathbf{s}\cdot\mathbf{A})$: *all* plates are of type $q$,
- $\Xi(\Lambda)$: collections of *polymers*: connected unions of $\frac{k}{2}\times\frac{k}{2}\times\frac{k}{2}$ cubes,
- $\phi^T$: *Mayer coefficient*,
- $K^{(\mathbf{s}\cdot\mathbf{A})}_{\Lambda,q}(X)$: *activity* of $X$.

# Cluster expansion

- Absolutely convergent expansion: $\exists \epsilon_k \to 0$ such that, for $m \geqslant 0$,

$$\sum_{\substack{\mathcal{X} \in \Xi(\Lambda) \\ |\mathcal{X}| \geq m}} \left| \phi^T(\mathcal{X}) \prod_{X \in \mathcal{X}} K_{\Lambda,q}^{(\mathbf{s} \cdot \mathbf{A})}(X) \right| \leq \epsilon_k^m$$