Non-perturbative renormalization group in a hierarchical Kondo model

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arXiv: 1506.04381 http://ian.jauslin.org/

Kondo model

- [N. Andrei, 1980]:

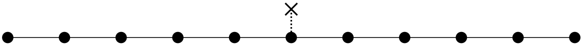
$$H = H_0 + V$$

  - H_0: kinetic term of the *electrons*

$$H_0 := \sum_x \sum_{\alpha=\uparrow,\downarrow} c_\alpha^\dagger(x) \left(-\frac{\Delta}{2} - 1 \right) c_\alpha(x)$$

  - V: interaction with the *impurity*

$$V = -\lambda_0 \sum_{j=1,2,3} \sum_{\alpha_1,\alpha_2,\alpha_3,\alpha_4} c^\dagger_{\alpha_1}(0)\sigma^j_{\alpha_1,\alpha_2} c_{\alpha_2}(0)\, d^\dagger_{\alpha_3} \sigma^j_{\alpha_3,\alpha_4} d_{\alpha_4}$$



Kondo effect: magnetic susceptibility

- Non-interacting magnetic susceptibility
  - Isolated impurity: $\chi^{(0)}(0,\beta) \underset{\beta\to\infty}{\longrightarrow} \infty$
  - Chain of electrons: $\lim_{\beta\to\infty} \lim_{L\to\infty} \frac{1}{L}\chi_e(0,\beta) < \infty.$
- Anti-ferromagnetic interaction: $\lambda_0 < 0$:

$$\lim_{\beta\to\infty} \chi^{(\lambda_0)}(0,\beta) < \infty.$$

- *Non-perturbative* effect: the qualitative behavior changes as soon as $\lambda_0 \neq 0$.

Previous results

- [J. Kondo, 1964]: third order Born approximation.

- [P. Anderson, 1970], [K. Wilson, 1975]: renormalization group approach

  - Sequence of effective Hamiltonians at varying energy scales.

  - For anti-ferromagnetic interactions, the effective Hamiltonians go to a *non-trivial fixed point*.

- `Remark`: [N. Andrei, 1980]: the Kondo model (suitably linearized) is exactly solvable via Bethe Ansatz.

Current results

- Hierarchical Kondo model: idealization of the Kondo model that shares its scaling properties.

- It is *exactly solvable*: the map relating the effective theories at different scales is *explicit* (no perturbative expansions).

- For $\lambda_0 < 0$, the flow tends to a non-trivial fixed point, and $\chi^{(\lambda_0)} < \infty$ in the $\beta \to \infty$ limit (Kondo effect).

Hierarchical beta function

- Beta function (*exact*)

$$C^{[m]} = 1 + 3(\ell_0^{[m]})^2 + 9(\ell_1^{[m]})^2 + 9(\ell_2^{[m]})^2 + 324(\ell_3^{[m]})^2$$

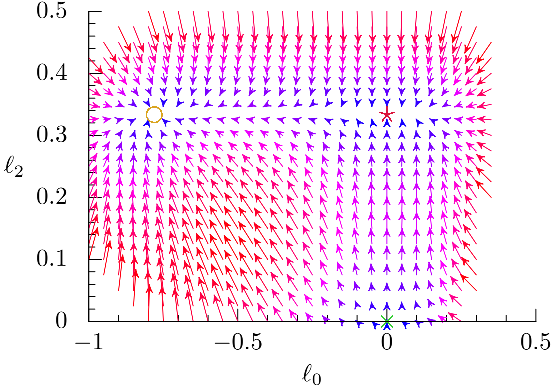
$$\ell_0^{[m-1]} = \frac{1}{C^{[m]}}\Big(\ell_0^{[m]} + 18\ell_0^{[m]}\ell_3^{[m]} + 3\ell_0^{[m]}\ell_2^{[m]} + 3\ell_0^{[m]}\ell_1^{[m]} - 2(\ell_0^{[m]})^2\Big)$$

$$\ell_1^{[m-1]} = \frac{1}{C^{[m]}}\Big(\frac{1}{2}\ell_1^{[m]} + 9\ell_2^{[m]}\ell_3^{[m]} + \frac{1}{4}(\ell_0^{[m]})^2\Big)$$

$$\ell_2^{[m-1]} = \frac{1}{C^{[m]}}\Big(2\ell_2^{[m]} + 36\ell_1^{[m]}\ell_3^{[m]} + (\ell_0^{[m]})^2\Big)$$

$$\ell_3^{[m-1]} = \frac{1}{C^{[m]}}\Big(\frac{1}{2}\ell_3^{[m]} + \frac{1}{4}\ell_1^{[m]}\ell_2^{[m]} + \frac{1}{24}(\ell_0^{[m]})^2\Big).$$

Flow



Fixed points: $\boldsymbol{\ell}^{(0)}$, $\boldsymbol{\ell}^{(+)}$, $\boldsymbol{\ell}^{*}$.

Kondo effect

- Fix $h = 0$.
- At $\boldsymbol{\ell}^{(+)}$, the susceptibility diverges as β.
- At $\boldsymbol{\ell}^{*}$, the susceptibility remains finite in the $\beta \to \infty$ limit.